Decay Instability of an Electromagnetic Wave Incident on a Weakly-Inhomogeneous Magnetised Plasma

Bhimsen K. Shivamoggi

Department of Theoretical Physics, Research School of Physical Sciences, The Australian National University, Canberra, Australia

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This paper considers the parametric decay of an electromagnetic wave incident on a weakly-inhomogeneous magnetised plasma into a Langmuir wave and an ion-acoustic wave. The Vlasov model is used in order to calculate the low-frequency electron-density perturbation produced by the beating of the side-band modes with the pump wave. The results show that the threshold value of the pump wave to undergo a decay instability drops in the presence of an inhomogeneity in the plasma.

1. Introduction

The parametric interaction of an intense coherent electromagnetic wave with collective modes in a plasma was investigated in detail by Drake et al. (1974) [1] in an attempt to provide an interpretation of observed phenomena in laser-produced plasmas and plasma-heating experiments. One of the consequences of the interaction of an incident (pump) electromagnetic wave with a plasma is the parametric excitation of two plasma waves. If the latter are both purely electrostatic, they are eventually absorbed in the plasma and this decay process then leads to enhanced (or anomalous) absorption of the incident electromagnetic wave. If one of the excited plasma waves is electromagnetic, it can escape from the plasma and show up as enhanced (or stimulated) scattering of the incident electromagnetic wave.

Experiments of Stamper et al. (1971) [2] showed that intense spontaneously-generated magnetic fields are present in laser-produced plasmas. These magnetic fields are usually strong enough to modify the spectrum of electrostatic modes in the plasma but not strong enough to influence the characteristics of propagation of the incident and the scattered electromagnetic modes. Further, all real plasmas are inhomogeneous, and in such plasmas the incident electromagnetic wave may get scattered off a low-frequency electrostatic drift wave. Therefore, Yu et al. (1974) [3] and Bujarbarua et al. (1976) [4] considered stimulated scattering of the incident electromagnetic wave off the drift-Alfven waves in an inhomogeneous magnetised plasma. The purpose of this paper is to consider cases wherein the plasma is weakly inhomogeneous and the incident electromagnetic wave decays instead into a Langmuir wave and an ion-acoustic wave to a first approximation.

Reprint requests to B. K. Shivamoggi, Department of Theoretical Physics, Research School of Physical Sciences, The Australian National University, Canberra, A.O.T. 2600, Australien.

2. Parametric Interaction of an Incident Electromagnetic Wave with Collective Modes in the Plasma

Consider a weakly-inhomogeneous plasma with a uniform background magnetic field $\boldsymbol{B}_0 = B_0 \hat{\boldsymbol{i}}_z$. The density gradient in the plasma is chosen along the x-axis. A large-amplitude, plane-polarised electromagnetic pump wave

$$\boldsymbol{E}_0 = 2 E_0 \hat{\boldsymbol{i}}_z \cos(\boldsymbol{k}_0 \cdot \boldsymbol{x} - \omega_0 t)$$

is incident on the plasma. The equilibrium state is comprised of electrons oscillating with high velocity in the incident electric field $\boldsymbol{E}_0$, with the ions remaining stationary and making up a neutralising background.

Let us perturb this equilibrium and study the time development of these perturbations using the linearised Vlasov-Maxwell equations. Consider a perturbation in the form of an electrostatic wave with frequency ω and wave number $\boldsymbol{k}$. This perturbation produces side-band modes at

$$\omega_\pm \equiv \omega \pm \omega_0\,, \quad \boldsymbol{k}_\pm \equiv \boldsymbol{k} \pm \boldsymbol{k}_0\,,$$

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for which one has

$$\left[\left(k_{\pm}^2 - \frac{\omega_{\pm}^2}{c^2}\varepsilon_{\pm}\right)\mathbf{I} - \boldsymbol{k}_{\pm}\boldsymbol{k}_{\pm}\right]\cdot \boldsymbol{E}_{\pm} = \frac{4\pi i}{c^2}\omega_{\pm}\boldsymbol{J}_{\pm}, \quad (1)$$

where $\boldsymbol{J}$ is the current density, ε the dielectric-response function, and

$$V_{\pm} \equiv V(\omega \pm \omega_0, \boldsymbol{k} \pm \boldsymbol{k}_0).$$

From the equation of motion of electrons, one obtains for $\omega_0/\omega_{p_e} \ll 1$ (or if ω_0/ω_{p_e} is arbitrary, but $k\lambda_{De} \ll 1$), and $\omega/\omega_0 \ll 1$ (here $\omega_{p_e}^2 = 4\pi n_0 e^2/m_e$, $\lambda_D = V_{Te}/\omega_{p_e}$, n_0 being the number density of electrons in the unperturbed state, m_e the mass of an electron and V_{Te} the electron thermal speed),

$$\boldsymbol{J}_{\pm} = \pm i(e^2/m_e\omega_0)\,\delta n_e(\boldsymbol{k},\omega)\,\boldsymbol{E}_{0\pm}, \quad (2)$$

where δn_e is the perturbation in the number density of electrons.

Using (2), eq. (1) gives upon inversion,

$$\boldsymbol{E}_{\pm} = -\boldsymbol{\omega}_{p_e}^2\frac{\delta n_e}{n_0}\left[\frac{(\mathbf{I} - \boldsymbol{k}_{\pm}\boldsymbol{k}_{\pm}/k_{\pm}^2)}{D_{\pm}} - \frac{\boldsymbol{k}_{\pm}\boldsymbol{k}_{\pm}}{k_{\pm}^2\omega_{\pm}^2\varepsilon_{\pm}}\right]\cdot \boldsymbol{E}_{0\pm}, \quad (3)$$

where

$$D_{\pm} \equiv k_{\pm}^2 c^2 - \omega_{\pm}^2\varepsilon_{\pm},$$
$$\omega_0^2 = \omega_{p_e}^2 + k_0^2 c^2.$$

In order to calculate the low-frequency electron-density perturbation $\delta n_e(\boldsymbol{k},\omega)$ produced by the beating of the sideband modes with the pump wave, Yu et al. (1974) and Bujarbarua et al. (1976) used the fluid model. In the following, we shall instead use the Vlasov model. Following Drake et al. (1974), let us first introduce the so-called ponderomotive force. The equation of motion of an electron in an inhomogeneous high-frequency field

$$\boldsymbol{E}_k(\boldsymbol{x})\exp\{-i\omega_k t\}$$

is

$$m_e\ddot{\boldsymbol{x}} = e\boldsymbol{E}_k(\boldsymbol{x})\exp\{-i\omega_k t\}. \quad (4)$$

Decompose the electron-motion into high-frequency and low-frequency components

$$\boldsymbol{x} = \boldsymbol{\xi} + \boldsymbol{R}, \quad (5)$$

where,

$$\boldsymbol{\xi} = -e\boldsymbol{E}_k/m_e\omega_k^2. \quad (6)$$

Using (5) and (6), Eq. (4) gives

$$m_e\ddot{\boldsymbol{R}} = e\boldsymbol{\xi}\cdot\nabla\boldsymbol{E}_k(\boldsymbol{R}) = -\frac{e^2}{2m_e\omega_k^2}\nabla(\boldsymbol{E}_k^2) = -\nabla\Psi, \quad (7)$$

where

$$\Psi = \frac{e^2}{m_e\omega_0^2}(\boldsymbol{E}_{0+}\cdot\boldsymbol{E}_- + \boldsymbol{E}_{0-}\cdot\boldsymbol{E}_+).$$

Note from Eq. (7) that the ponderomotive force on the ions is smaller than that on the electrons by the mass ratio and is therefore negligible.

Using Eq. (7), the linearised drift kinetic equation (which is obtained by averaging the Vlasov equation over the gyro-radius) becomes

$$\frac{\partial f}{\partial t} + \boldsymbol{V}\cdot\nabla f - \frac{1}{m_e}(e\nabla\Phi + \nabla\Psi)\frac{\partial f}{\partial \boldsymbol{V}} - c\frac{(e\nabla\Phi + \nabla\Psi)\times\boldsymbol{B}_0}{B_0^2}\nabla f = 0, \quad (8)$$

where

$$\boldsymbol{E} \equiv -\nabla\Phi.$$

One obtains from eq. (8), upon Fourier analysis,

$$\delta n_e = \frac{k^2}{4\pi e}\left[\Phi(\boldsymbol{k},\omega) + \frac{\Psi(\boldsymbol{k},\omega)}{e}\right] \times\left[\left(1 - \frac{\hat{\omega}_e}{\omega}\right)\chi_e(\boldsymbol{k},\omega) + \frac{\hat{\omega}_e}{\omega k^2\lambda_{D_e}^2}\right], \quad (9)$$

where

$$\hat{\omega}_s \equiv V_{T_s}^2\frac{\boldsymbol{k}\cdot(\boldsymbol{B}_0\times\nabla n_0)}{(2B_0^2|e_s|/m_s c)}, \quad \lambda_{D_s}^2 \equiv \frac{V_{T_s}^2}{\omega_{p_s}^2}$$

and for a Maxwellian plasma,

$$\chi_e(\boldsymbol{k},\omega) = \frac{1}{k^2\lambda_{D_e}^2}\left[1 + \frac{\omega}{kV_{T_e}}Z\left(\frac{\omega}{kV_{T_e}}\right)\right],$$

$$Z(\zeta) \equiv \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-x^2}}{x-\zeta}\,\mathrm{d}x, \quad (10)$$

and V_{Te} is the thermal speed of electrons. Similarly, one has for the ions

$$\delta n_i = -\frac{k^2}{4\pi e}\,\Phi(\boldsymbol{k},\omega)\left[\left(1+\frac{\hat{\omega}_i}{\omega}\right)\chi_i(\boldsymbol{k},\omega)-\frac{\hat{\omega}_i}{\omega\,k^2\lambda_{D_i}^2}\right]. \tag{11}$$

Using (9) and (11), Poisson's equation

$$-k^2\Phi(\boldsymbol{k},\omega) = 4\pi\sum e_s\,\delta n_s \tag{12}$$

gives

$$\Phi(\boldsymbol{k},\omega) = -\frac{\Psi(\boldsymbol{k},\omega)}{e\,\varepsilon(\boldsymbol{k},\omega)}\left[\chi_e(\boldsymbol{k},\omega)+\frac{\hat{\omega}_e}{\omega}\left(-\chi_e+\frac{1}{k^2\lambda_{D_e}^2}\right)\right], \tag{13}$$

where

$$\varepsilon(\boldsymbol{k},\omega) \equiv 1+\chi_e(\boldsymbol{k},\omega)+\chi_i(\boldsymbol{k},\omega)+\frac{\hat{\omega}_e}{\omega}\left(-\chi_e+\frac{1}{k^2\lambda_{D_e}^2}\right)+\frac{\hat{\omega}_i}{\omega}\left(\chi_i-\frac{1}{k^2\lambda_{D_i}^2}\right). \tag{14}$$

Using (13), (9) gives

$$\delta n_e = -\frac{k^2}{4\pi m_e\omega_0^2}\,\frac{\left[\chi_e+\frac{\hat{\omega}_e}{\omega}\left(-\chi_e+\frac{1}{k^2\lambda_{D_e}^2}\right)\right]}{\varepsilon(\boldsymbol{k},\omega)}\left[1+\chi_i+\frac{\hat{\omega}_i}{\omega}\left(\chi_i-\frac{1}{k^2\lambda_{D_i}^2}\right)\right]$$
$$\times\,(\boldsymbol{E}_{0+}\cdot\boldsymbol{E}_-+\boldsymbol{E}_{0-}\cdot\boldsymbol{E}_+)\,. \tag{15}$$

From (3) and (15), one obtains the dispersion relation

$$\frac{1}{\chi_e+\frac{\hat{\omega}_e}{\omega}\left(-\chi_e+\frac{1}{k^2\lambda_{D_e}^2}\right)}+\frac{1}{1+\chi_i+\frac{\hat{\omega}_i}{\omega}\left(\chi_i-\frac{1}{k^2\lambda_{D_i}^2}\right)}$$
$$= k^2\left[\frac{|\boldsymbol{k}_-\times\boldsymbol{V}_0|^2}{k_-^2\,D_-}-\frac{(\boldsymbol{k}_-\cdot\boldsymbol{V}_0)^2}{k_-^2\,\omega_-^2\,\varepsilon_-}+\frac{|\boldsymbol{k}_+\times\boldsymbol{V}_0|^2}{k_+^2\,D_+}-\frac{(\boldsymbol{k}_+\cdot\boldsymbol{V}_0)^2}{k_+^2\,\omega_+^2\,\varepsilon_+}\right], \tag{16}$$

where

$$\boldsymbol{V}_0 \equiv e\boldsymbol{E}_0/m_e\omega_0\,.$$

Note that (16) describes the parametric coupling of a low-frequency electrostatic mode $(\boldsymbol{k},\omega)$ and two side bands $(\boldsymbol{k}_\pm,\omega_\pm)$.

3. Decay Instability of the Incident Electromagnetic Wave

Consider a pump wave with $\omega_0\approx\omega_{pe}$. Then the high-frequency side bands are predominantly electrostatic (Drake et al., 1974), and so (16) becomes

$$\frac{1}{\left(1-\frac{\hat{\omega}_e}{\omega}\right)\chi_e+\frac{\hat{\omega}_e/\omega}{k^2\lambda_{D_e}^2}}+\frac{1}{1+\left(1+\frac{\hat{\omega}_i}{\omega}\right)\chi_i-\frac{\hat{\omega}_i/\omega}{k^2\lambda_{D_i}^2}} \approx -(\boldsymbol{k}\cdot\boldsymbol{V}_0)^2\left[\frac{1}{\omega_-^2\,\varepsilon_-}+\frac{1}{\omega_+^2\,\varepsilon_+}\right]. \tag{17}$$

Now note, that

$$\chi_e \approx \begin{cases} -\dfrac{\omega_{p_e}^2}{\omega^2}\,(1+3k^2\lambda_{D_e}^2+\dfrac{i\sqrt{\pi}}{k^2\lambda_{D_e}^2}\,\dfrac{\omega}{k\,V_{T_e}}\exp-\{\omega^2/k^2V_{T_e}^2\}\,, & \dfrac{\omega}{k}\gg V_{T_e} \\ \dfrac{1}{k^2\lambda_{D_e}^2}\left(1+i\sqrt{\pi}\,\dfrac{\omega}{k\,V_{T_e}}\right), & \dfrac{\omega}{k}\ll V_{T_e} \end{cases}$$

$$\chi_i \approx \begin{cases} -\dfrac{\omega_{p_i}^2}{\omega^2} + \dfrac{i\sqrt{\pi}}{k^2 \lambda_{D_i}^2} \dfrac{T_e}{T_i} \dfrac{\omega}{k V_{Ti}} \exp\{-\omega^2/k^2 V_{T_i}^2\}, & \dfrac{\omega}{k} \gg V_{T_i}, \\ \dfrac{1}{k^2 \lambda_{D_i}^2}\left(1 + i\sqrt{\pi}\dfrac{\omega}{k V_{T_i}}\right), & \dfrac{\omega}{k} \ll V_{T_i}. \end{cases} \tag{18}$$

Considering

$$\left|\frac{\omega_\pm}{k_\pm}\right| \gg V_{T_e}, \quad \frac{\omega}{\omega_0} \ll 1, \quad \frac{\hat{\omega}_e}{\omega_{p_e}} \ll 1,$$

one obtains, on using (18),

$$\omega_\mp^2 \, \varepsilon_\mp \approx \mp 2\,\omega_{p_e}(\omega \mp \Delta + i\Gamma_1), \tag{19}$$

where

$$\Delta \equiv \omega_0 - \omega_e, \quad \omega_e^2 \equiv \omega_{p_e}^2 (I + 3k^2 \lambda_{D_e}^2),$$

$$\Gamma_1 \equiv \sqrt{\pi}\frac{(\omega_{p_e} - \hat{\omega}_e)}{2k^3 \lambda_{D_e}^3} \exp\{-1/k^2 \lambda_{D_e}^2\}.$$

Using (19), (18) gives

$$\frac{1}{\left[\left(1 - \dfrac{\hat{\omega}_e}{\omega}\right)\chi_e + \dfrac{\hat{\omega}_e/\omega}{k^2 \lambda_{D_e}^2}\right]} + \frac{1}{\left[\left(1 + \dfrac{\hat{\omega}_i}{\omega}\right)\chi_i - \dfrac{\hat{\omega}_i/\omega}{k^2 \lambda_{D_i}^2}\right]} \approx \frac{(\boldsymbol{k}\cdot \boldsymbol{V}_0)^2}{2\,\omega_{p_e}(\omega - \Delta + i\Gamma_1)}, \tag{20}$$

where we have considered the case $\varepsilon_- \approx 0$.

Considering $V_{T_e} \gg \omega/k \gg V_{T_i}$, and using (18), (20) becomes

$$k^2 \lambda_{D_e}^2 \left[1 - i\sqrt{\pi}\frac{(\omega - \hat{\omega}_e)}{k V_{T_e}}\right] + \frac{1}{-\left(1 + \dfrac{\hat{\omega}_i}{\omega}\right)\dfrac{\omega_{p_i}^2}{\omega^2} - \dfrac{\hat{\omega}_i/\omega}{k^2 \lambda_{D_i}^2}} \approx \frac{(\boldsymbol{k}\cdot \boldsymbol{V}_0)^2}{2\,\omega_{p_e}(\omega - \Delta + i\Gamma_1)} \tag{21}$$

or

$$\left[\frac{\omega^2}{1 + \dfrac{\hat{\omega}_i}{\omega}\left(I + \dfrac{\omega^2}{k^2 V_{T_i}^2}\right)} + 2i(\omega - \hat{\omega}_e)\Gamma_2 - k^2 C_s^2\right](\omega - \Delta + i\Gamma_1) + \frac{(\boldsymbol{k}\cdot \boldsymbol{V}_0)^2 \omega_{p_i}^2}{2\,\omega_{p_e}} = 0, \tag{22}$$

where

$$C_s^2 = \frac{2kT_e}{m_i}, \quad \Gamma_2 = \sqrt{\frac{\pi}{8}}\, k C_s \sqrt{\frac{m_e}{m_i}}.$$

Put

$$\omega = k C_s + i\gamma \tag{23}$$

and let

$$\omega_0 \approx k C_s + \omega_e, \quad \hat{\omega}_i / k C_s \ll 1 \tag{24}$$

which describes decay of the incident wave into a Langmuir wave and an ion-acoustic wave. Then (22) gives (assuming $T_e/T_i \gg I$).

$$\left[\gamma + \left(1 - \frac{\hat{\omega}_e}{k C_s}\right)\Gamma_2\right]\left[\gamma + \left(1 - \frac{\hat{\omega}_i}{k V_{T_i}}\right)\Gamma_1\right] = \Omega_0^2, \tag{25}$$

where

$$\Omega_0^2 \equiv (\boldsymbol{k} \cdot \boldsymbol{V}_0)^2 \, \omega_{\mathrm{p_i}}^2 / 4 \, k \, C_{\mathrm{S}} \, \omega_{\mathrm{p_e}} .$$

From (25), the threshold value for the pump wave to undergo a parametric decay into a Langmuir wave and an ion-acoustic wave is given by

$$(\Omega_0^2)_{\mathrm{th}} = \left(1 - \frac{\hat{\omega}_{\mathrm{e}}}{k \, C_{\mathrm{S}}}\right) \left(1 - \frac{\hat{\omega}_{\mathrm{i}}}{k \, V_{\mathrm{T_i}}}\right) \Gamma_1 \Gamma_2 . \quad (26)$$

(26) shows that the threshold value of the pump wave to undergo a decay instability drops in the presence of an inhomogeneity in the plasma.

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